

More Iverson Brackets

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Abstract

In this very short note, we summarize the counterparts of the Iverson brackets in selected areas of mathematics.

Keywords

Iverson brackets; notation.

Introduced in [2, p. 11], the Iverson brackets convert a logical statement into numerical values according to:

$$\llbracket P \rrbracket = \begin{cases} 1, & \text{if } P \text{ is true,} \\ 0, & \text{if } P \text{ is false,} \end{cases} \quad (1)$$

where $\llbracket \cdot \rrbracket$ denotes the Iverson brackets and P is a true-false proposition. A comprehensive and convincing exposition of the utility of the Iverson brackets is given in [3]. As shown in [1], the Iverson brackets can substantially improve the manipulation of expressions involving summations and products. In this one-page letter, we summarize the counterparts of the Iverson brackets in other areas of mathematics. Table 1 lists the results.

The adopted notation is: k is a discrete index; $P(k)$ is a predicate (or property) of k ; a_k are complex numbers; b_k are integer numbers; Q_k are propositions; $\neg$ is the logical negation operation; $\emptyset$ is the empty set; the superscript C denotes the set complement operation; the logical *or* and the logical *and* operations are denoted by $\vee$ and $\wedge$, respectively; symbols $\cup$ and $\cap$ are the set union and the set intersection operations, respectively; the least common multiple and the greatest common divisor of integers m and n are $m \vee n \triangleq \text{lcm}(m, n)$, and $m \wedge n \triangleq \text{gcd}(m, n)$ respectively.

Table 1: Iverson bracket for selected areas of mathematics

Area	Iverson Brackets	Sum	Product
Arithmetic	$\llbracket P \rrbracket = \begin{cases} 1, & \text{if } P \text{ is true,} \\ 0, & \text{if } P \text{ is false.} \end{cases}$	$\sum_{P(k)} a_k = \sum_k a_k \cdot \llbracket P(k) \rrbracket$	$\prod_{P(k)} a_k = \prod_k a_k^{\llbracket P(k) \rrbracket}$
Logic	$\llbracket P \rrbracket = P$	$\bigvee_{P(k)} Q_k = \bigvee_k Q_k \wedge \llbracket P(k) \rrbracket$	$\bigwedge_{P(k)} Q_k = \bigwedge_k Q_k \vee \llbracket \neg P(k) \rrbracket$
Set Theory	$\llbracket P \rrbracket = \begin{cases} \emptyset^C, & \text{if } P \text{ is true,} \\ \emptyset, & \text{if } P \text{ is false.} \end{cases}$	$\bigcup_{P(k)} A_k = \bigcup_k A_k \cap \llbracket P(k) \rrbracket$	$\bigcap_{P(k)} A_k = \bigcap_k A_k \cup \llbracket \neg P(k) \rrbracket$
Divisibility Theory	$\llbracket P \rrbracket = \begin{cases} 1, & \text{if } P \text{ is true,} \\ 0, & \text{if } P \text{ is false.} \end{cases}$	$\bigvee_{P(k)} b_k = \bigvee_k b_k^{\llbracket P(k) \rrbracket}$	$\bigwedge_{P(k)} b_k = \bigwedge_k b_k \cdot \llbracket P(k) \rrbracket$

References

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